Dynamics of Planetary Systems

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Outline
formation and evolution timescales
planet masses
planetary dynamical architecture
long term orbital evolution of planets
small bodies
moons and rings
spin-orbit dynamics
tidal evolution
From the solar nebula hypothesis, our solar system is thought to have formed from the collapse of a molecular cloud.

**Formation timescales**

The “solar nebula” hypothesis posits that planetary systems form from the collapse of an interstellar molecular cloud.

- **molecular cloud**: $\sim 10^4$ yr
- **protostellar disk**: $\sim 10^5$ yr
- **protoplanetary disk**: $\sim 10^6$ yr
- **giant planets+debris disk**: $\sim 10^8$ yr
- **mature planetary system**

How do we know the timescales?

astronomical observations

“proplyds” in the Orion Nebula

age <~10 myr
How do we know the timescales?

astronomical observations

beta Pictoris
distance 63 ly
age 12 my
planet-b 7 mJ
How do we know the timescales?

radiometric ages: meteorites

Radiometric age-dating of meteorite samples

Oldest SS solids are ~4.567 Gyr old
How do we know the timescales?

radiometric ages: lunar samples

Oldest lunar minerals are ~4.44 Gyr old

Cathodoluminescence image of a 400-μm Jack Hills zircon.

J. Valley, U. Wisconsin
How do we know the timescales?

radiometric ages: terrestrial samples

Oldest terrestrial minerals are ~4.375 Gyr old
How do we know the timescales?

geological markers

Impact crater chronology
- a tool to age-date geological history
Planetary system size scales

Four or Five distinct neighborhoods in the Solar system

also some stragglers in-between (NEOs, Centaurs)
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Four or Five distinct neighborhoods in the Solar system

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Planetary system diversity

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Earth-mass planets around a neutron star

Upsilon Andromedae A System

TRAPPIST 1


NASA, ESA, and A. Feild (STScI)
Planetary system diversity

Earth-mass planets around a neutron star

Upsilon Andromedae A System

TRAPPIST 1
Planet masses

Confirmed exoplanets with measured masses
(total number: 617)
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Distribution of planet masses: theoretical (Malhotra, 2015)

log-log plot of PDF of (planet mass/earth-mass)

slope = -0.6

slope = -1.2
All objects in the Solar System move along elliptical paths with the Sun at one of the two foci of the ellipse. The shape of the ellipse can be completely determined by:

- **a**: semimajor axis
- **e**: eccentricity

\[
ed = \left(1 - \frac{b^2}{a^2}\right)^{\frac{1}{2}}
\]

Orbits:
- Circular: \( e = 0 \)
- Elliptic: \( 0 < e < 1 \)
- Parabolic: \( e = 1 \)
- Hyperbolic: \( e > 1 \)

- **f**: true anomaly of the body
- **E**: eccentric anomaly
- **M**: mean anomaly
  \[ M = E - e \sin E = nt \]  
  with \( n \) is the orbital frequency
  \[ n = \sqrt{\frac{GM}{a^3}} \]

To characterize the orientation of the ellipse in space, with respect to an arbitrary orthogonal reference frame centered on the Sun, we need to introduce three additional angles:

- **i**: inclination of the orbital plane
- **Ω**: longitude of the ascending node
- **ω**: argument of the perihelion

For the case where \( i=0 \) and/or \( e=0 \), it's useful to introduce:

- **ω**: longitude of perihelion
  \[ \omega = \omega + \Omega \]
- **κ**: mean longitude
  \[ \kappa = M + \omega \]
Planetary system dynamical stability

planets expected to emerge from their birth disks in nearly co-planar well-separated orbits

how large a departure from co-planarity?

\[ \Delta i \sim \left( \frac{m}{3M} \right)^{1/3} \]

orbital separations?

\[ \Delta a \gtrsim 3.5 \left( \frac{(m_1+m_2)}{3M} \right)^{1/3} a \]
Planetary system dynamical stability

Hill Radius = \((m/3M)^{1/3} a\)
Orbital Period Ratios of adjacent planets

(multiple-planet Kepler systems data from Fabrycky et al., 2014)
Orbital Period Ratios of adjacent planets

Malhotra, 2015

Broad peak at 1.5–2.5, drop-off at smaller and larger values

(multiple-planet Kepler systems data from Fabrycky et al., 2014)
Orbital Period Ratios of adjacent planets

Malhotra, 2015

(multiple-planet *Kepler* systems data from Fabrycky et al., 2014)
Dimensionless orbital separation

\[ D = \frac{a_{\text{out}} - a_{\text{in}}}{\frac{1}{2}(a_{\text{out}} + a_{\text{in}})} \]

\[ = 2(P^{\frac{2}{3}} - 1)/(P^{\frac{2}{3}} + 1) \]

\[ P = P_{\text{outer}}/P_{\text{inner}} \]

by Kepler's 3rd Law

\[ P_j^2 \sim a_j^3 \]
Planetary system dynamical stability

Two planets

minimum orbital separation is \( \sim 3.46 \) times mutual Hill radius

\[
D = 2\sqrt{3}\left(\frac{m_1 + m_2}{3m_*}\right)^{\frac{1}{3}}
\]

G.W. Hill, 1878
Gladman, 1993
Planetary system dynamical stability

\(N > 2\) planets

no analytical criterion

empirical: generalize Hill’s criterion

\[
D = K \left( \frac{m_1 + m_2}{3m_*} \right)^{\frac{1}{3}}
\]

\(K > 3.46\)… but by how much?
likely depends upon planet multiplicity (N),
eccentricities (‘angular momentum deficit’, AMD),
planet mass ratios (m1/m2),
age of the system (‘dynamical age’, t/T1)
Planetary system dynamical stability

N>2 planets
no analytical criterion
empirical: generalize Hill’s criterion

\[ D = K \left( \frac{m_1 + m_2}{3m_*} \right)^{\frac{1}{3}} \]

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age of the system (‘dynamical age’, t/T_1)

Ansatz: K is a random variate
Distribution of $K$ (orbital separation in units of mutual Hill radius)

$Lissauer$ et al (2011)
$m = M_\odot (R/R_\odot)^{2.06}, R > R_\odot$

$m = M_\odot (R/R_\odot)^{3}, R < R_\odot$

Weiss & Marcy (2014)

PDF (log $K$)

Solar system

Malhotra, 2015

Look to Solar System
& Kepler multis with use of mass-radius relationship(s)
Distribution of $K$ (orbital separation in units of mutual Hill radius)

Look to Solar System & Kepler multis with use of mass-radius relationship(s)

Lissauer et al (2011)
Weiss & Marcy (2014)

Mass-radius relationship:

$m = M_\oplus (R/R_\oplus)^{2.06}, R > R_\oplus$
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Malhotra, 2015

Solar system

PDF (log $K$)

Solar system

Typical $K \sim 12-15$
i.e. $\Delta a \sim (12-15) R_H$
but significant dispersion!
Distribution of orbital eccentricities?
poorly known… (but potentially very important for habitability)
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Distribution of orbital eccentricities? poorly known… (but potentially very important for habitability)

A theoretical prediction based on numerical simulations (Hansen & Murray (2013), and an analytical 1-D ergodic model for the giant impact phase of terrestrial planet formation (Tremaine, 2015)

... predictions somewhat higher than observed
Long term evolution

Dynamical instabilities — collisions — consolidation

Numerical simulations of Kepler-system analogs (Volk & Gladman, 2015)
Long term evolution
Secular dynamics

Two planets — example of Jupiter and Saturn
timescale $\sim P(4 \left(\frac{a_2}{a_1}\right)^2 \frac{M_*}{m})$

Murray & Dermott, 1999
Long term evolution
Secular dynamics

Earth’s orbital plane and orbital eccentricity

quasi-periodic climate forcing (Milankovitch cycles)

Murray & Dermott, 1999
Small bodies — debris belts
Asteroid Belt

quasi-stable … long term source of planetary impactors

rich dynamics: collisions, orbital drift due to solar radiation forces and resonances/chaotic diffusion
Kuiper Belt
source of Centaurs, Jupiter-family comets

Current observational census of KB - June 2018

- subject to heavy observational biases
- most discoveries closer than 50 au heliocentric distance

Total number known: 1871

Dynamical classes
“Classical”
“Resonant”
“Scattered”
“Detached”
Oort Cloud

source of long period comets
potential for exchange with other planetary systems
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